Rational sinc interpolants and point shifts

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The interpolation of functions with steep gradients is greatly improved by putting more points in the vicinity of these gradients. In pseudospectral methods, a conformal map of the domain is used for that purpose and classically introduced into the polynomials replacing the functions appearing in the differential equation. The exponential convergence is conserved, when the interpolation/collocation points are zeros or extrema of orthogonal polynomials. However, because of the use of the chain rule, this leads to complicated differentiation matrices.

To avoid the latter, the first two authors have suggested in 2001 to use a linear rational barycentric interpolant introduced in [1] instead of the usual polynomial one when solving a differential equation on an interval. Differentiation formulas as simple as those of the interpolating polynomial [2] lead to systems that are themselves as simple as those of the classical polynomial pseudospectral method [3]. The effect of the maps is impressive [4,5].

The first author has studied the effect of conformal shifts in the (Fourier) periodic case [6].

In the present work we treat the approximation on the infinite line, replacing the sinc interpolant with a limit of linear rational sinc ones, and we show that, here as well, the exponential convergence is conserved with the conformal map. We also demonstrate through numerical examples how point shifts greatly improve the interpolant's accuracy for the approximation of functions with steep gradients. Moreover, since we start with equispaced points instead of Chebyshev ones, the precision is even better than with the linear rational interpolant at conformally shifted Chebyshev nodes mentioned above.

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