RBF techniques for reconstruction in FVM

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Abstract: Many physics and engineering problems give rise to time-dependent partial differential equations (PDEs), and one of the most significant and extensively used among them is the hyperbolic conservation law. One characteristic aspect of conservation laws is the potential development of discontinuities, known as shocks, or sharp fronts in the solution, even when the initial condition is smooth. These discontinuities can lead to non-physical oscillations in the numerical solution. The goal of numerical techniques is to accurately capture these discontinuities while avoiding oscillations and achieving solutions with a high order of convergence. Over the past few decades, the finite volume method (FVM) has emerged as a widely-used numerical technique for solving conservation laws. FVM has a rich history and has evolved into various forms and methodologies. These methods are either linear or non-linear and share similarities with finite difference schemes for hyperbolic PDEs, but are also adaptable to unstructured grids. According to the Godunov theorem, linear schemes must be either first-order accurate or oscillatory. To achieve higher accuracy without oscillations, non-linear schemes have been developed. These schemes typically employ Godunov's approach, artificial viscosity, or high-order reconstruction.

The earliest attempts at achieving higher than first-order reconstructions date back to the application of flux and slope limiter methods to obtain second-order accurate schemes in FVM. Subsequently, higher-order Essentially Non-Oscillatory (ENO) reconstructions were developed and widely used for approximating hyperbolic PDEs in computational fluid dynamic problems. ENO reconstruction involves determining sets of stencils surrounding a control volume, computing a reconstruction on each stencil, and selecting the smoothest reconstruction for the control volume. Weighted ENO (WENO) reconstruction improves upon this by using a weighted sum of different reconstructions based on their smoothness.

This talk focuses on the development of kernel approximation methods, particularly radial basis function (RBF) approximations, in the reconstruction step of FVM. After a brief review of RBF-WENO techniques, we introduce a novel non-oscillatory RBF reconstruction that captures shocks and sharp fronts effectively using a single central stencil. This approach,

referred to as FVM-WSR, employs a single central stencil alongside a weighted smoothed reconstruction (WSR) technique to suppress non-physical oscillations near shocks or sharp fronts. Unlike WENO, which uses multiple stencils, FVM-WSR requires only one stencil, making it suitable for unstructured meshes and adaptable for various kernel-based interpolations. It is applicable to unstructured meshes and can be adapted for various kernel-based interpolations. To ensure stability and accuracy, we employ polyharmonic spline basis functions and use their scalability property to construct stable local interpolation matrices. In the temporal domain, we use strong stability-preserving Runge-Kutta (SSPRK) methods. This work was originally done in collaboration with Navid Soodbakhsh from the University of Isfahan.