On stability and convergence of a domain decomposition FE/FD method for Maxwell's equations

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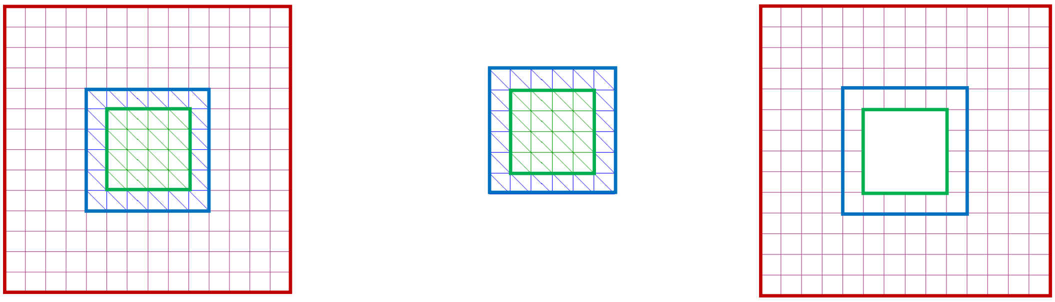
Abstract

Stability and convergence analyses for the domain decomposition finite element/finite difference (FE/FD) method are presented. The analyses are developed for semi-discrete finite element schemes for the time-dependent Maxwell's equations. In this setting explicit finite element schemes and corresponding domain decomposition algorithms are constructed. Several numerical examples validat convergence rates obtained in the theoretical analysis. Our basic model is given in terms of the electric field $E\left(x,t\right)=\left(E_1,E_2,E_3\right)\left(x,t\right)$, for $x\in\mathbb{R}^3$, $t\in[0,T]$. Under certain assumptions, the Cauchy problem for the Maxwell's equations for electric field $E\left(x,t\right)$ in non-conductive media is given as

$$\begin{cases} \varepsilon(x)\dfrac{\partial^2 E}{\partial t^2}+\nabla\times\nabla\times E=0, \\ \nabla\cdot(\varepsilon E)=0, \\ E(x,0)=f_0(x), \qquad \dfrac{\partial E}{\partial t}(x,0)=f_1(x),\ \ x\in\mathbb{R}^3,\ \ t\in(0,T]. \end{cases} \tag{0.1}$$

To solve the problem (0.1) numerically, we restrict the spatial domian $\mathbb{R}^3$ to a bounded convex Domain $\Omega\subset\mathbb{R}^3$, with boundary $\partial\Omega$. Then a domain decomposition, hybrid, finite element/finite difference scheme is constructed in certain split of the domain Ω. More specifically, we divide the computational domain Ω into two subregions, $\Omega_{\rm FEM}$ and $\Omega_{\rm FDM}$ such that $\Omega=\Omega_{\rm FEM}\cup\Omega_{\rm FDM}$ and $\Omega_{\rm FEM}$ is a subset of the convex hul of $\Omega_{\rm FDM}$. The function $\varepsilon(x)$ in equation (0.1) is known in $\Omega_{\rm FDM}$ and need to be determined only in $\Omega_{\rm FEM}$. When solving the inverse problem of reconstruction of unknown dielectric permittivity function ε in $\Omega_{\rm FEM}$, such configuration allows stable computation of the unknown $\varepsilon(x)$, even if it has large discontinuities in $\Omega_{\rm FEM}$. The communication between $\Omega_{\rm FEM}$ and $\Omega_{\rm FDM}$ is arranged using a mesh overlapping through a two-element thick layer around $\Omega_{\rm FEM}$ as shown by blue and green common boundaries in Figure 1. The blue boundary is outer boundary of $\Omega_{\rm FEM}$ and inner boundary of $\Omega_{\rm FDM}$. Similarly, the green boundary is the inner boundary of $\Omega_{\rm FEM}$, from which the solution is copied to the green boundary of $\Omega_{\rm FDM}$.

The common nodes of both Ω_{FEM} and Ω_{FDM} domains belong to either of the following boundaries (see Figure 1). Then the main loop in time for the explicit hybrid FEM/FDM scheme, that solves (0.1), associated with appropriate boundary conditions, at each time step, is described in Algorithm 1 below:



a) $\Omega = \Omega_{\mathrm{FEM}} \cup \Omega_{\mathrm{FDM}}$ b) Ω_{FEM} c) Ω_{FDM}

Figure 1: *Domain decomposition and mesh discretization in Ω. The domain Ω is a combination of the quadrilateral finite difference mesh Ω_{FDM} presented on c), and the finite element mesh Ω_{FEM} presented on b). Domains Ω_{FEM} and Ω_{FDM} overlaps by two layers of structured nodes such that they have common boundaries outlined by green and blue colors.*

Algorithm 1 Domain decomposition process for hybrid FE/FD scheme

1: On the mesh Ω_{FDM}, where FDM is used, update the Finite Difference (FD) solution.
2: On the mesh Ω_{FEM}, where FEM is used, update the Finite Element (FE) solution.
3: Copy FE solution obtained at nodes $\omega_{\diamond}$ (nodes on the green boundary of Figure 1) as a boundary condition on the inner boundary for the FD solution in Ω_{FDM}.
4: Copy FD solution obtained at nodes ω_{o} (nodes on the blue boundary of Figure 1) as a boundary condition for the FE solution on $\partial\Omega_{FEM}$ of Ω_{FEM}.
5: Apply boundary condition at $\partial\Omega$ at the red boundary of Ω_{FDM}.

References

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